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Non-leptonic B decays into a charmed tensor meson

J.-P. Lee[a]

Department of Physics and IPAP, Yonsei University, Seoul, 120-749, Republic of Korea

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Abstract. In the framework of factorization and the heavy quark effective theory, $B \to D_2^* \pi$ modes are analyzed. We adopt the result from the QCD sum rule calculation for the hadronic matrix elements at leading order of $\Lambda_{\rm QCD}/m_Q$ and $\alpha_{\rm s}$. The QCD sum rule results are well compatible with the current data, with the prediction for the branching ratios $\mathcal{B}(\bar{B}^0 \to D_2^{*+}\pi^-) = 8.94 \times 10^{-4}$ and $\mathcal{B}(B^- \to D_2^{*0}\pi^-) = 9.53 \times 10^{-4}$ for $N_C^{\rm eff} = 2$. We give constraints on the interception $\tau(1)$ and the slope parameter ρ^2 of the leading Isgur–Wise function from the experimental bounds. It is argued that the observation of non-zero $\mathcal{B}(\bar{B}^0 \to D_2^{*0}\pi^0)$ directly measures the non-factorizable effects.

1 Introduction

The advent of the B-factory era in KEK and SLAC opens new possibilities for the study of very suppressed B decays. Non-leptonic two-body decays into tensor (T) mesons, which are among them, deserve much attention nowadays. Experimental data on them provides only upper bounds for the branching ratios. Two-body hadronic B decays involving a tensor meson T in the final state has long been studied [1–3] using the non-relativistic quark model of Isgur, Scora, Grinstein, and Wise (ISGW) [4] with the factorization ansatz. Their predictions for the branching ratios are rather small, while the preliminary results from the Belle Collaboration indicate that the branching ratios for $B \to PT$ (P = pseudoscalar) may not be very small compared to the $B \to PP$ modes [5]. Recently, both charmed and charmless $B \to P(V)T$ (V = vector) decays were updated [6] using the ISGW2 model [7], which is an HQET-based improvement of the original model. They are in many respects complementary to $B \to P(V)$ decays. From a theoretical point of view, a more reliable description of the $B \to T$ transition is required.

The biggest obstacle in theoretical predictions is the hadronic matrix elements. The factorization hypothesis is a widely accepted assumption. Recently, it was pointed out that the factorization parameter a_2 is process dependent and that there is a non-zero strong phase difference between color-allowed and color-suppressed decay modes, based on the first observation of $\bar{B}^0 \to D^{(*)0}\pi^0$ by Belle and CLEO [8,9]. It is thus very interesting to see what happens in $B \to D_2^*\pi$. The charmed tensor D_2^* is a member of a doublet $(1^+, 2^+)$ with $j_l^P = 3^+/2$, while (D_0', D_1') corresponds to $(0^+, 1^+)$ with $j_l^P = 1^+/2$. The neutral decay mode $\bar{B}^0 \to D_2^{*0}\pi^0$ is dominated by the color-suppressed internal W-emission diagram. Within the factorization approach (we will not consider the small contributions from the W-exchange diagram for simplicity), the decay amplitude is proportional to $\langle 0|V - A|T\rangle$. It can easily be shown, however, that such a factorized term vanishes [3]. This is a great advantage of tensor mesons in the final state because the decay amplitude is greatly simplified. Given the factorization assumption, therefore, the hadronic uncertainties are condensed to the $B \to T$ transition matrix elements. On the other hand, the observation of a sizable branching ratio for $\bar{B}^0 \to D_2^{*0}\pi^0$ would provide direct information on the non-factorizable effects. This is another benefit of studying the production of tensor mesons.

Present bounds from the experiments are [10]

$$\mathcal{B}(B^+ \to \bar{D}_2^{*0}\pi^+) < 1.3 \times 10^{-3} , \quad \mathcal{B}(B^+ \to \bar{D}_2^{*-}\pi^+) < 2.2 \times 10^{-3} . \tag{1}$$

In this paper, we analyze the two-body B decays into a charmed tensor D_2^*. Heavy quark effective theory (HQET) is an appropriate framework in this process. In HQET, the $B \to D_2^*$ transition matrix element is parameterized by one universal Isgur–Wise (IW) function at leading order of $\Lambda_{\rm QCD}/m_Q$, where m_Q is the heavy quark mass. An extensive study of the leading and subleading IW function in semileptonic decays is found in [11]. In general, the IW function depends on the velocity transfer $y \equiv v \cdot v'$ where $v(v')$ is the four-velocity of $B(D)$. Typically, the kinematically allowed range of $y - 1$ is very small in the $B \to D$ transition. It is customary to parameterize the IW function $\tau(y)$ in terms of its interception $\tau(1)$ and the slope parameter ρ^2, and expand in $(y-1)$. The branching ratio is directly proportional to $|\tau(1)|^2$. Unlike the groundstate to groundstate transition, the heavy quark symmetry (HQS) does not guarantee the normalization of $\tau(1)$ in $B \to D_2^*$.

[a] e-mail: jplee@phya.yonsei.ac.kr

This is because at zero recoil ground- to excited-state transition is suppressed by $\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$, due to the HQS.

Though the HQET is very economic and allows us to make a systematic expansion of $\Lambda_{\rm QCD}/m_Q$, one still needs some non-perturbative methods to evaluate the IW function. In the present work, we adopt the QCD sum rule results for the $B \to D_2^*$ leading IW function [12]. The QCD sum rule is among the most reliable non-perturbative methods [13]. It takes into account the non-trivial QCD vacuum which is parameterized by various vacuum condensates in order to describe the non-perturbative nature. In the QCD sum rule approach, hadronic observables can be calculated by evaluating two- or three-point correlation functions. The hadronic currents for constructing the correlation functions are expressed by the interpolating fields. In describing the excited D mesons of the $(1^+, 2^+)$ states, the transverse covariant derivative is included in the interpolating fields [12].

In the next section, the decay amplitudes are given within the factorization, and the QCD sum rule results for the leading IW function are summarized. Section 3 contains the numerical results and discussions. The QCD sum rule results are compared with the ISGW2 model predictions. Possible next-to-leading order corrections are also discussed. We give a summary in Sect. 4.

2 Hadronic matrix elements and QCD sum rules

The effective weak Hamiltonian for $B \to D_2^* \pi$ is

$$\mathcal{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} V_{cb} V_{ud}^* \Big[c_1(\mu)(\bar{d}u)(\bar{c}b) + c_2(\mu)(\bar{c}u)(\bar{d}b) + \cdots \Big] , \qquad (2)$$

where $(\bar{q}_1 q_2) = \bar{q}_1 \gamma^\mu (1-\gamma_5) q_2$, and $c_i(\mu)$ are the Wilson coefficients.

Within the factorization framework, the decay rate amplitudes are given by

$$\mathcal{A}_{+-} \equiv \mathcal{A}(\bar{B}^0 \to D_2^{*+}\pi^-) = \mathcal{T} + \mathcal{E} , \qquad (3a)$$

$$\mathcal{A}_{00} \equiv \mathcal{A}(\bar{B}^0 \to D_2^{*0}\pi^0) = \frac{1}{\sqrt{2}}(-\mathcal{C} + \mathcal{E}) , \qquad (3b)$$

$$\mathcal{A}_{0-} \equiv \mathcal{A}(B^- \to D_2^{*0}\pi^-) = \mathcal{T} + \mathcal{C} , \qquad (3c)$$

where

$$\mathcal{T} = \frac{G_{\rm F}}{\sqrt{2}} V_{cb} V_{ud}^* \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle \times \langle D_2^{*+} | (\bar{c}b)_{V-A} | \bar{B}^0 \rangle a_1 , \qquad (4a)$$

$$\mathcal{C} = \frac{G_{\rm F}}{\sqrt{2}} V_{cb} V_{ud}^* \langle D_2^{*0} | (\bar{c}u)_{V-A} | 0 \rangle \times \langle \pi^0 | (\bar{d}b)_{V-A} | \bar{B}^0 \rangle a_2 , \qquad (4b)$$

$$\mathcal{E} = \frac{G_{\rm F}}{\sqrt{2}} V_{cb} V_{ud}^* \langle \bar{B}^0 | (\bar{d}b)_{V-A} | 0 \rangle \times \langle D_2^{*0}\pi^0 | (\bar{c}u)_{V-A} | 0 \rangle a_2 , \qquad (4c)$$

are the color-allowed external W-emission, color-suppressed internal W-emission and W-exchange amplitudes, respectively. Note that (3) satisfies the isospin triangle relation

$$\mathcal{A}_{+-} = \sqrt{2}\mathcal{A}_{00} + \mathcal{A}_{0-} . \qquad (5)$$

The factorized matrix elements are parameterized as

$$\langle 0 | A^\mu | P \rangle = {\rm i} f_P p_P^\mu , \qquad (6a)$$

$$\begin{aligned} \langle T | j^\mu | B \rangle = {} & {\rm i} h(m_P^2) \epsilon^{\mu\nu\rho\sigma} \epsilon^*_{\nu\alpha} p_B^\alpha (p_B + p_T)_\rho (p_B - p_T)_\sigma \\ & + k(m_P^2) \epsilon^{*\mu\nu} (p_B)_\nu \\ & + \epsilon^*_{\alpha\beta} p_B^\alpha p_B^\beta [b_+(m_P^2)(p_B + p_T)^\mu \\ & + b_-(m_P^2)(p_B - p_T)^\mu] , \end{aligned} \qquad (6b)$$

where $j^\mu = V^\mu - A^\mu$ and V^μ (A^μ) denote a vector (an axial-vector) current. Here f_P denotes the decay constant of the relevant pseudoscalar meson, and $h(m_P^2)$, $k(m_P^2)$, $b_+(m_P^2)$, $b_-(m_P^2)$ express the form factors for the $B \to T$ transition.

We will neglect the internal W-exchange diagram $\mathcal{E}$ for simplicity. And the color-suppressed internal W-emission diagram $\mathcal{C}$ is forbidden, because [3]

$$\mathcal{C} \sim \langle 0 | j^\mu | T \rangle \sim p_\nu \epsilon^{\mu\nu}(p) + p^\mu \epsilon^\nu_\nu(p) = 0 . \qquad (7)$$

Thus, the decay mode $\bar{B}^0 \to D_2^{*0}\pi^0$ is not allowed in the factorization scheme.

In the heavy quark limit where $m_Q \to \infty$, all the form factors are expressed by one universal Isgur–Wise (IW) function $\tau(y \equiv v \cdot v')$ [12]:

$$h(y) = \frac{\tau(y)}{2m_B\sqrt{m_B m_T}} , \qquad (8a)$$

$$k(y) = \sqrt{\frac{m_T}{m_B}}(1+y)\tau(y) , \qquad (8b)$$

$$b_+(y) = -\frac{\tau(y)}{2m_B\sqrt{m_B m_T}} , \qquad (8c)$$

$$b_-(y) = \frac{\tau(y)}{2m_B\sqrt{m_B m_T}} , \qquad (8d)$$

where v (v') denotes the four-velocity of B (T).

At this point, non-perturbative methods are needed to evaluate $\tau(y)$. We use the QCD sum rule results at leading order of $\Lambda_{\rm QCD}/m_Q$. The QCD sum rule result for $\tau(y)$ is [12]

$$\begin{aligned} & \tau(y) f_{-,1/2} f_{+,3/2}\, {\rm e}^{-(\bar{\Lambda}_{-,1/2} + \bar{\Lambda}_{+,3/2})/M} \\ & = \frac{1}{2\pi^2(y+1)^3} \int_0^{\omega_c} {\rm d}\omega_+ \omega_+^3 {\rm e}^{-\omega_+/M} \\ & \quad - \frac{1}{12} m_0^2 \frac{\langle \bar{q}q \rangle}{M} - \frac{1}{3 \times 2^5 \pi} \langle \alpha_{\rm s} GG \rangle \frac{y+5}{(y+1)^2} , \end{aligned} \qquad (9)$$

where

Table 1. Branching ratios for $B \to D_2^* \pi$ with QCD sum rule (ISGW2 [6]). The QCD sum rule results are from the linear approximation of (11)

	$\xi = 0.1$	$\xi = 0.3$	$\xi = 0.5$
$\mathcal{B}_{+-} \times 10^4$	11.42 (3.11)	10.14 (2.76)	8.94 (2.44)
$\mathcal{B}_{0-} \times 10^4$	12.17 (3.31)	10.81 (2.94)	9.53 (2.59)

$$f_{-,1/2}^2 \,\mathrm{e}^{-2\bar{\Lambda}_{-,1/2}/M}$$
$$= \frac{3}{16\pi^2}\int_0^{\omega_{c0}} \omega^2 \mathrm{e}^{-\omega/M}\mathrm{d}\omega - \frac{1}{2}\langle\bar{q}q\rangle\left(1-\frac{m_0^2}{4M^2}\right) , \quad (10a)$$
$$f_{+,3/2}^2 \,\mathrm{e}^{-2\bar{\Lambda}_{+,3/2}/M} \quad (10b)$$
$$= \frac{1}{2^6\pi^2}\int_0^{\omega_{c2}} \omega^4 \mathrm{e}^{-\omega/M}\mathrm{d}\omega - \frac{1}{12}m_0^2\langle\bar{q}q\rangle - \frac{1}{2^5\pi}\langle\alpha_s GG\rangle M ,$$

from which $f_{\pm,3/2(1/2)}$ and $\bar{\Lambda}_{\pm,3/2(1/2)}$ are determined. Here $\langle\bar{q}q\rangle$ and $\langle\alpha_s GG\rangle$ are vacuum condensates, $m_0^2 = 0.8\ \mathrm{GeV}^2$, and M is the Borel parameter. The continuum thresholds ω_c, ω_{c0}, and ω_{c2} are adjustable parameters for the numerical analysis. Typically, the kinematically allowed range of y is very narrow, $1 \le y \lesssim 1.3$. It is therefore convenient to approximate $\tau(y)$ linearly as

$$\tau(y) = \tau(1)[1-\rho^2(y-1)] . \quad (11)$$

The QCD sum rule predicts from (9) and (10)

$$\tau(1) = 0.74 \pm 0.15 , \quad \rho^2 = 0.90 \pm 0.05 . \quad (12)$$

The errors come from the QCD sum rule parameters, the continuum threshold and the Borel parameter.

3 Results and discussions

The decay rate for $B \to PT$ is in general given by

$$\Gamma(B \to PT) = \frac{|\vec{p}_P|^5}{12\pi m_T^2}\left(\frac{m_B}{m_T}\right)^2 \left|\frac{\mathcal{A}(B\to PT)}{\epsilon^*_{\mu\nu}p_B^\mu p_B^\nu}\right|^2 , \quad (13)$$

where $\vec{p}_P$ is the pseudoscalar three momentum. The branching ratios for $B \to D_2^*\pi$ are summarized in Table 1 with the abbreviations

$$\mathcal{B}_{+-} \equiv \mathcal{B}(\bar{B}^0 \to D_2^{*+}\pi^-) , \quad (14a)$$
$$\mathcal{B}_{0-} \equiv \mathcal{B}(B^- \to D_2^{*0}\pi^-) . \quad (14b)$$

In Table 1, we give the numerical results for various $\xi \equiv 1/N_C^{\mathrm{eff}}$, where N_C^{eff} is the effective number of color, while it is reported that $N_C^{\mathrm{eff}}(B \to D\pi) \approx 2$ [14]. For comparison, results from the ISGW2 model [6] are also given. The QCD sum rule predicts about four-times larger branching ratios.

Figure 1 shows the transition form factors as functions of y for both the QCD sum rule and the ISGW2, where the form factors $F^{B\to T}$ are defined by

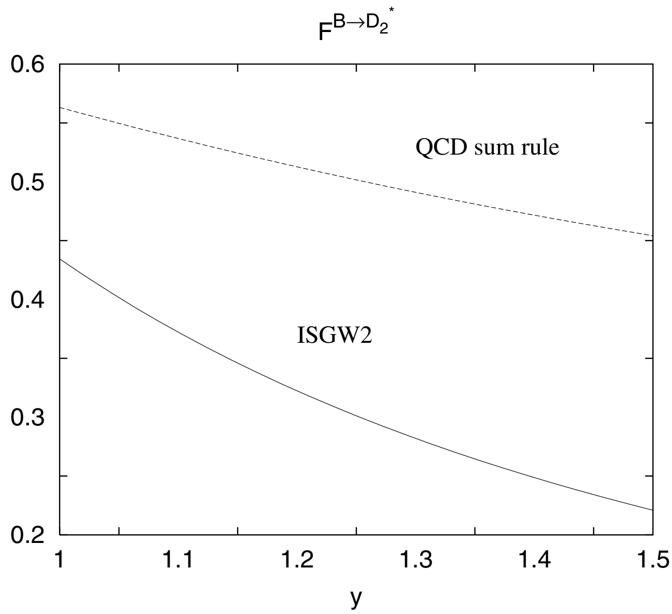

Fig. 1. Form factors $F^{B\to D_2^*}(y)$ from QCD sum rule and ISGW2

$$\mathcal{T} = \mathrm{i}\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^* f_P \epsilon^*_{\mu\nu}p_B^\mu p_B^\nu \times F^{B\to T}(m_P^2)a_1 , \quad (15a)$$
$$F^{B\to T}(m_P^2) = k(m_P^2) + (m_B^2 - m_T^2)b_+(m_P^2) + m_P^2 b_-(m_P^2) . \quad (15b)$$

With the linear approximation of (11) and (12), the ratio of the hadronic and semileptonic branching ratios is

$$\frac{\mathcal{B}(B\to D_2^*\pi)}{\mathcal{B}(B\to D_2^*\ell\bar{\nu})} = 0.20 . \quad (16)$$

In this fraction, the common factor of $|\tau(1)|^2$ is cancelled and only the slope parameter ρ^2 remains at leading order. Near future experiments will check this ratio. One interesting thing is that the value is very close to that of the $B \to D_1$ transition [15]:

$$\frac{\mathcal{B}(B^-\to D_1^0\pi^-)}{\mathcal{B}(B^-\to D_1^0\ell^-\bar{\nu})} = 0.21 \pm 0.08. \quad (17)$$

On the other hand, conversely, the experimental bounds on the branching ratios can constrain $\tau(1)$ and ρ^2. Figure 2 shows the allowed region (shaded) of these two parameters from the given branching ratios. Note that the central value of the QCD sum rule result $(\tau(1), \rho^2) = (0.74, 0.90)$ resides very close to the boundary. In addition, the measured value of the ratio (16) will determine ρ^2 regardless of $\tau(1)$. More precise measurements will thus provide a simple test of the leading order description of $B \to D_2^*$.

Now consider the next-to-leading order (NLO) corrections to the above analysis. The NLO contributions come from both $\Lambda_{\mathrm{QCD}}/m_Q$ and α_s. In HQET, $\Lambda_{\mathrm{QCD}}/m_Q$ corrections appear in a two-fold way. At the Lagrangian level,

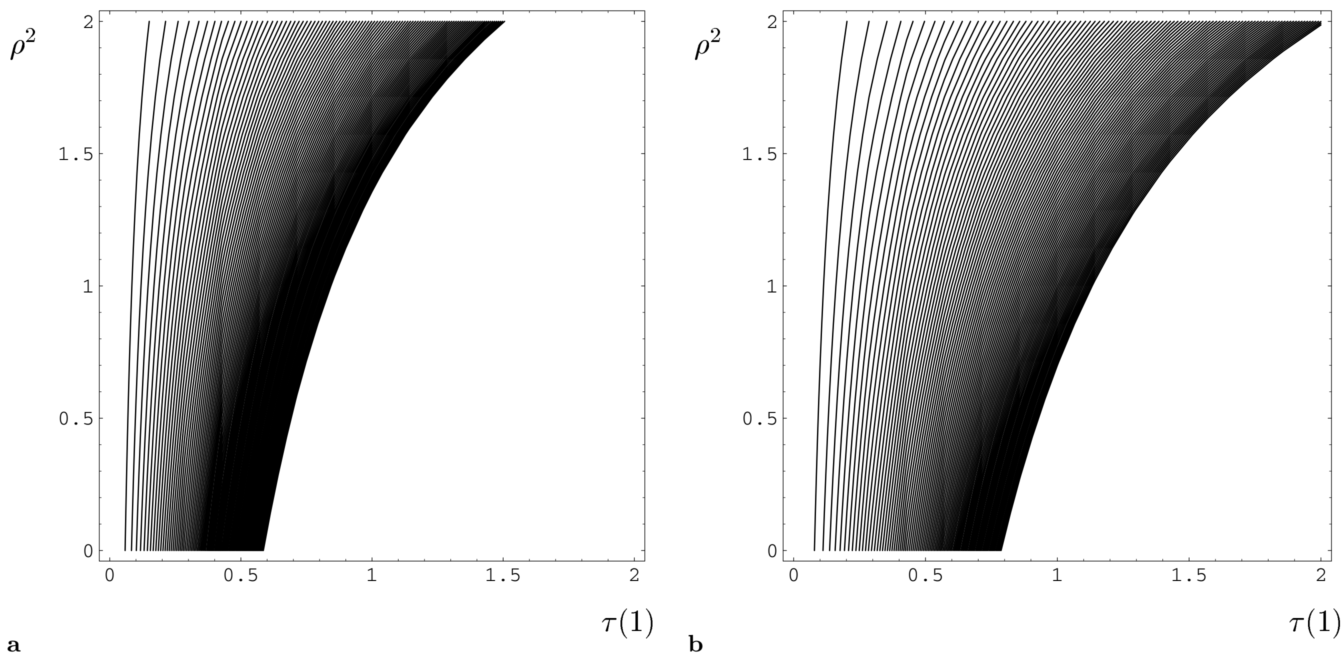

Fig. 2a,b. Allowed regions (shaded) of $(\tau(1), \rho^2)$ from the experimental bounds for **a** $B^- \to D_2^{*0}\pi^-$ and **b** $\bar{B}^0 \to D_2^{*+}\pi^-$

subleading terms are summarized in λ_1 and λ_2. It is a usual convention that λ_1 parameterizes the kinetic term of the higher order derivative, while λ_2 represents the chromomagnetic interaction which explicitly breaks the heavy quark spin symmetry. Their effects are known to be small [11]. At the current level, $\Lambda_{\rm QCD}/m_Q$ corrections come from the current matching procedure onto the effective theory. The correction terms originate from the small portion of the heavy quark fields which corresponds to the virtual motion of the heavy quark. During the current matching, eight subleading IW functions are newly introduced and two of them are independent.

Compared with the $B \to D_1$ transition, $\Lambda_{\rm QCD}/m_Q$ corrections to $B \to D_2^*$ would be rather small. This is due to the tensor structure in the final state. At zero recoil (properties at this point are very important because the kinematically allowed region is quite narrow around it), the transition matrix is proportional to

$$\langle D_2^*(v,\epsilon)|j^\mu|B(v)\rangle \sim \epsilon^{\mu\nu} v_\nu = 0 \ . \tag{18}$$

As argued briefly in Sect. 1, the vanishing matrix element is well explained by the HQS at the heavy quark limit. On the other hand, for the $B \to D_1$ transition at zero recoil,

$$\langle D_1(v,\epsilon)|j^\mu|B(v)\rangle \sim \epsilon^{*\mu} \ , \tag{19}$$

which can be non-zero in general. That is the reason why NLO corrections are more important in $B \to D_1$.

Another NLO contribution from $\mathcal{O}(\alpha_{\rm s})$ corrections is studied in $B \to D_0', D_1'$ decays in [16]; it remains as a good challenge in the $B \to D_2^*$ process.

Finally, if there were observed a sizable value of $\mathcal{B}_{00} \equiv \mathcal{B}(\bar{B}^0 \to D_2^{*0}\pi^0)$, it should come from non-factorizable effects. Or indirectly, the measurement of $(\kappa\mathcal{B}_{+-} - \mathcal{B}_{0-})$ where $\kappa \equiv \tau_{B^+}/\tau_{\bar{B}^0} \approx 1.07$ is the B life-time ratio would test the validity of the isospin relation (5) and the general factorization scheme. The values in Table 1, of course, satisfy the relation $\mathcal{B}_{0-}/\mathcal{B}_{+-} = \kappa$.

4 Summary

Using the QCD sum rule results for the leading IW function of $B \to D_2^*$, we investigated the non-leptonic two-body decays $B \to D_2^*\pi$ within the framework of factorization. The predicted branching ratios are about four times larger than the recent calculations based on the ISGW2. The study of the tensor meson is very advantageous because the decay amplitudes are simple, and some of the decay modes are directly related to the non-factorizable effects. Present B factories lead to a big optimism of producing copious tensor mesons, and more precise and reliable theoretical works are requested. The NLO analysis of both $\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$ and $\mathcal{O}(\alpha_{\rm s})$, in this respect, is challenging and will improve the theoretical reliability.

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